Aspects of holography and irrelevant deformations

Lecture 1:

effective string theory

References:

Polchinski - Strominger '91, 1203.1054

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Irrelevant deformations of 2d CFTs (mostly $T\bar{T}$)

- Provide us with novel examples of nonlocal but UV-complete theories.
- These deformations are universal and share many features of quantum gravity (QG), i.e. they're 2d toy models of QG
- Holographically, they can be used to study 3d gravity in a box (with a finite cutoff)
- They also lead to new models of holography for asymptotically flat spacetimes (relevant for the RW) and warped AdS (relevant for BHs)

Before we discuss (or even define) this kind of deformations, let us first consider the "simplest toy model of QG": an infinitely long bosonic free string

Shares many features with QG:

- minimal length (theory is nonlocal)
- universal time delay ($\Delta t = l_s^2 \epsilon$)
- lack of local off-shell observables*

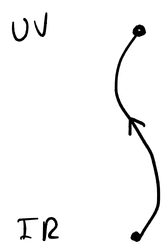
In particular, this model is nonrenormalizable but

- features a "good" S-matrix valid in the UV
- exhibits a new kind of RG flow: asymptotic fragility.

The last two features are particularly interesting. For example in GR:

$$S_{EH} = \frac{1}{16\pi G}\int d^4x \, R \sim \int d^4x \left(\partial h \, \partial h + \underbrace{l_p \, h \, \partial h \, \partial h + l_p^2 \, h^2 \, \partial h \, \partial h + \dots}_{\text{nonrenormalizable interactions}} \right)$$

Diferent ways to fix NR theories like GR:



- add new dofs (as in string theory)
- hope for the best (asymptotic safety)

- The infinitely long string does not flow to a UV fixed point.
- It is the simplest example of a $T\bar{T}$ deformed QFT.

The infinitely long string

Polchinski-Strominger (PS): the standard quantization of the string cannot be used to describe an $\underbrace{\text{effective string}}_{\text{e.g. QCD flux tubes, cosmic strings}}$ in D-dimensional Minkowski.

Let $X^\mu(\sigma, \tau)$ denote the embedding of the string in D-dimensional Minkowski spacetime.

Physical excitations of the string: D-2 dofs. On the other hand

- lightcone quantization : broken Lorentz invariance if $D \neq 26$
- covariant quantization : D-1 dofs if $D \neq 26$
- noncritical strings : $D-2+\underbrace{1}_{\text{Liouville mode}}$ dofs

The PS term

PS added a nonlocal term to the action to make it $\overbrace{\text{conformal}}^{\text{reparametrization invariance}}$

$$S = \frac{1}{4\pi}\int d^2\sigma \left\{ \underbrace{\frac{1}{l_s^2}\,\partial_+ X^\mu \partial_- X_\mu}_{\text{Polyakov string}} + 2\beta \underbrace{\frac{\partial_+^2 X^\mu \partial_- X_\mu \; \partial_+ X^\nu \partial_-^2 X_\nu}{(\partial_+ X^\mu \partial_- X_\mu)^2}}_{\text{Unique term with weight (1,1) up to total derivatives, constraints}} + \dots \right\}$$

l_s^0 l_s^2

Conventions: (τ, σ) : worldsheet coordinates

X^μ : target space coordinates, $\mu = 0, \dots, D-1$

(∂_+, ∂_-) : $\left(\frac{\partial\tau + \partial\sigma}{2}, \frac{\partial\tau - \partial\sigma}{2}\right)$

The PS term

PS added a nonlocal term to the action to make it $\overbrace{\text{conformal}}^{\text{reparametrization invariance}}$

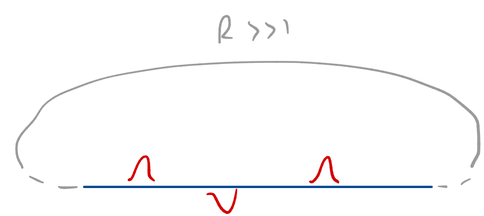
$$S=\frac{1}{4\pi}\int d^2\sigma\left\{\underbrace{\overset{R^2}{\frac{1}{l_s^2}\,\partial_+X^\mu\,\partial_-X_\mu}}_{\text{Polyakov string}}\;\overset{(\text{no } R^{-1})}{+}\;2\beta\,\underbrace{\overset{R^{-2}}{\frac{\partial_+^2X^\mu\,\partial_-X_\mu\;\partial_+X^\nu\,\partial_-^2X_\nu}{(\partial_+X^\mu\,\partial_-X_\mu)^2}}}_{\text{Unique term with weight (1,1) up to total derivatives, constraints}}\;\overset{+\,\mathcal{O}(R^{-4})}{+\;\dots}\right\}$$

The PS term becomes local when expanded around the long string background:

$$X^\mu=X^\mu_{cl}+\underset{\text{perturbations}}{Y^\mu}$$

$X^0_{cl}=R\tau$

$X^1_{cl}=R\sigma$



It is not difficult to check that (use $\delta X^\mu = -\epsilon_+ \partial_+ X^\mu - \frac{\beta l_s^2}{2} \partial_+^2 \epsilon_+ \frac{\partial_- X^\mu}{\partial_+ X^\mu \partial_- X_\mu} + \mathcal{O}(R^{-3})$)

$$T(z)\,T(0) = \underbrace{\frac{D + 12\beta}{2 z^4}}_{\beta = \frac{26-D}{12} \;\Rightarrow\; c_T = c_p + c_{PS} + c_g = 0} + \frac{2T(0)}{z^2} + \frac{\partial T(0)}{z} + \mathcal{O}(R^{-2})$$

Another consequence of the PS term (use the Virasoro constraints)

$$M^2 = \underbrace{\frac{R}{2 l_s^2}}_{\text{long string}} - \frac{D-2}{12 R} + \mathcal{O}(R^{-2})$$

↳ $D-2$ dofs !

<u>Conclusion</u>: the effective field theory (EFT) description of the string is valid for any D, features $D-2$ dofs, and nonrenormalizable terms suppressed by l_s.

EFT of the long string in static gauge

The PS action has gauge redundancy: D number of X^μ fields but only $D-2$ dof. In particular it features an infinite number of nonrenormalizable terms that must be determined order by order in l_s.

An alternative approach is to consider the static gauge where $X^0 = \tau$, $X^1 = \sigma$.

In this case we can think of the long string as a system of $D-2$ Goldstone bosons that realize $\underbrace{\text{D-dimensional Poincaré nonlinearly}}_{\text{the symmetries that are spontaneously broken by the string}}$.

The infinitely long string breaks the Poincaré group $ISO(1, D-1)$ to

$$ISO(1,1) \otimes SO(D-2)$$

$\downarrow$ Lorentz symmetry of the worldsheet

$\downarrow$ "flavour" group of $D-2$ Goldstones X^i

The spontaneously broken symmetries are the $D-2$ translations transverse to the string and # boosts. $\left(\delta^{\alpha i}_{\epsilon} X^j = -\epsilon \left(\delta^{ij} \sigma^\alpha + X^i \partial^\alpha X^j \right) \right)$

Goldstone's theorem still applies to spacetime symmetries but, since boosts are coordinate-dependent translations, we only have $D-2$ Goldstone fields.

Callan - Coleman - Wess - Zumino

Consider D fields $X^\mu = (\tau, \sigma, X^i(\tau,\sigma))$ that nonlinearly realize the Poincaré symmetry so that the X^i transform as $X^i \to X^i + x_0^i$. Then, the Lorentz invariant Lagrangian is the sum of geometric invariants built out of

$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$:

$$S = - \int d^2\sigma \, \underbrace{\sqrt{-h} \, \{ l_s^{-2}}_{\substack{\text{the Nambu-Goto (NG) action} \\ \text{(in static gauge):}}} + \underbrace{\alpha_0^{-1} (K^i_{\alpha\beta})^2 + \dots \}}_{\substack{\text{coefficients of the effective} \\ \text{action determined from experiment.} \\ \text{We ignore these!}}}$$

$$S_{NG} = - \frac{1}{l_s^2} \int d^2\sigma \sqrt{-h}$$

The NG action

The NG action is a valid IR description of the string in any D valid for scales much larger than l_s. This action is clearly nonrenormalizable:

$$S_{NG} = -\frac{1}{4 l_s^2} \int d^2\sigma \left[\# \, \partial_\alpha X^i \partial^\alpha X^i + c_2 \left(\partial_\alpha X^i \partial^\alpha X^i \right)^2 + c_3 \underbrace{\left(\partial_\alpha X^i \partial_\beta X^i \partial^\alpha X^j \partial^\beta X^j \right)}_{} + \dots \right.$$

How do we fix these coefficients? ($c_2 = -1/2$, $c_3 = 1$)

Why only terms w/ derivatives?

At high energies, the additional geometric invariants we have omitted become important and the $D \neq 26$ theory need be UV completed appropriately.

At $D = 26$ the NG theory is UV-complete by itself and hence equivalent to the Polyakov string.*

The NG action

In contrast to the usual treatment of the NG action, here we are assuming there are no other fields than the $D-2$ X^i fields – there is no B-field and no dilaton.

In particular we are considering a free string with $g_s = 0$.

The $g_s = 0$ theory turns out to be nontrivial. We will show this by finding the non-perturbative S-matrix. Before we do this we must consider two puzzles:

1. Where is the PS term? Why haven't we added it to the action?

2. Can we define an S-matrix for massless fields in 2d?

Perturbative analysis

Why can't we add the PS term to the NG action. There are two reasons:

1. The PS term is nonlocal in $h_{\alpha\beta}$ and in fact is given by

$$S_P = \frac{26-D}{192\pi} \int d^2\sigma \sqrt{-h}\, R \frac{1}{\Box} R$$

→ In the noncritical string this term comes from the measure.

2. The PS term breaks the nonlinearly realized boosts!

It must be the case that the effect of the PS term is already included in the NG action.

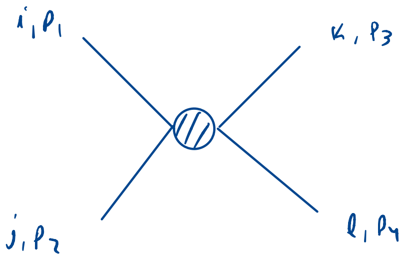
We look for the PS term in $2 \to 2$ scattering. Note that since the PS term is of $\mathcal{O}(\ell_s^0)$, it corresponds to a one-loop effect. (Also true of $R\frac{1}{\Box}R$ of NC string).

The SO(D-2) flavour symmetry of the X^i fields restrict the scattering amplitude to

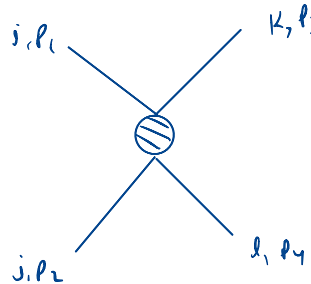
$$M_{ij,kl} = A\,\delta_{ij}\delta_{kl} + B\,\delta_{ik}\delta_{jl} + C\,\delta_{il}\delta_{jk}$$

$i \quad j \quad k \quad l$

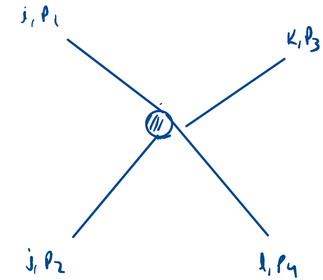
$p_1 \quad p_2 \quad p_3 \quad p_4$



$s = -(p_1+p_2)^2$



$t = -(p_1+p_3)^2$



$u = -(p_1+p_4)^2$

Crossing symmetry: $A(s,u,t) = A(s,t,u) = B(t,s,u) = C(u,t,s)$

We consider first the contribution to $O(l_s^2)$ to the amplitude. Loop corrections are of order $O(l_s^n)$, $n \geq 4$.

The relevant part of the NG action for tree and one-loop contributions is:

$$S_{NG} = -\frac{1}{4 l_s^2}\int d^2\sigma \left[\# \, \partial_\alpha X^i \partial^\alpha X^i + c_2 \left(\partial_\alpha X^i \partial^\alpha X^i\right)^2 + c_3 \left(\partial_\alpha X^i \partial_\beta X^i \partial^\alpha X^j \partial^\beta X^j\right) \right]$$

<u>Tree level</u>: A quick computation reveals that

$$A_{tree} = -\frac{l_s^2}{4}\left[\underbrace{(c_3 + 2c_2)}_{0} s^2 - 2 c_3 \underbrace{t u}_{0} \right]$$

Note that in 2d we have (hint: use lightcone variables and $s+t+u=0$)

$$t = 0 \Rightarrow u = -s \qquad \text{or} \qquad u = 0 \Rightarrow t = -s$$

So in 2d we find that

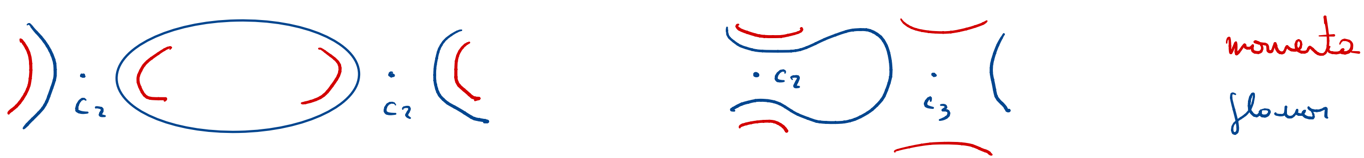
$$A = 0 \,, \qquad \text{no annihilations at tree level !}$$

In $d>2$ the crossing symmetry would imply $B=C=0$. In 2d we have:

$$M_{ijkl} = -\frac{l_s^2}{2}\left(\delta_{ik}\,\delta_{jl}\,\underbrace{su}_{-s^2} + \delta_{il}\,\delta_{jk}\,\underbrace{st}_{-s^2}\right)$$

This result is universal (valid for any D) and will be useful later.

<u>One-loop</u>: The one-loop calculation receives contributions of several "fish" diagrams:



$\sim D-2$ independent of D

We see that the different topologies can lead to special values of D!

<u>A puzzle</u>: in dimensional regularization we have divergences which require the addition of counterterms. The only terms at $\mathcal{O}(l_s^4 s^3)$ compatible with the linearly realized $ISO(1,1) \otimes SO(D-2)$ symmetries are

$$\underbrace{\partial_\beta X^j \partial_\gamma X^i \partial_\alpha \partial^\beta X^i \partial^\alpha \partial^\gamma X^j}_{\text{comes from } \int d^2\sigma \sqrt{-h}\, R} \quad , \quad \underbrace{(\partial_\alpha \partial_\beta X^i \partial_\gamma X^i)^2}_{\text{the SP term } \int d^2\sigma\, R \frac{1}{\Box} R}$$

The Ricci scalar is a total derivative in 2d but it must be included in dim reg where it helps us regulate divergences:

$$S_{ct} = -\frac{D-8}{48\pi\epsilon} \int d^2\sigma \sqrt{-h}\, R$$

Actually the divergent amplitude has $A\alpha\, stu$ which vanishes in 2d exactly (meaning $B=C=0$)

After a lengthy calculation we find that the finite part of A is given by:

$$A_{1\text{-loop}} = -\frac{l_s^2}{192\pi}\left\{ (D-26)\, s^3 + stu\left[\frac{16D+4}{3} - 2(D-8)\log\frac{-s}{\mu^2}\right] + 12\,tu\left(t\log\frac{s}{t} + u\log\frac{s}{u}\right)\right\}$$

this is exactly the contribution of the SP term

this vanishes exactly in 2d, i.e. $A = stu = 0 \Rightarrow B = C = 0$

this vanishes in 2d but leads to $B, C \neq 0$

This result contains many lessons:

- From the point of view of EFT, we see that $D=26$ is special since $A=0$, i.e. there are no annihilations of the NG string to $O(l_s^4)$. This is a hint of the integrability of the NG action in $D=26$, which will be discussed in the next lecture. ($D=3$ is also special since in this case the full amplitude is proportional to $M_{\text{one-loop}} \propto s^3+t^3+u^3 = 0$ so there are no interactions at $O(l_s^4)$)

- The effect of the PS term is already included in the NG action, but it appears at one-loop.

Here, while the PS term appears as a nonlocal interaction in the Wilsonian

$h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$ is conformally flat

action in the "conformal gauge", it appears as a nonlocal interaction in the

1PI action in the static gauge.

- Interestingly, we find that the log terms disappear the amplitude to $O(l_s^4)$

so that it becomes purely polynomial. For example for the B-term in $D=26$

in the "t-channel" with $t=0$ we have:

$$B_{1\text{-loop}} = -\frac{l_s^4}{192\pi}\left[(D-26)\,t^3 + 12\,s\,u\left(s\log\frac{t}{s+i0^+} + u\log\frac{t}{u+i0^+}\right)\right] = \frac{i\,l_s^4}{16}\,s^3$$