Aspects of holography and irrelevant deformations

Lecture 10:

$T\bar{T}$ and Jackiw-Teitelboim gravity II

References:

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The dressed S-matrix

In the previous lecture, we coupled a Poincaré-invariant QFT to flat Jackiw-Teitelboim gravity

$$S = \int_\Sigma d^2\sigma \sqrt{|g|}\,(\phi R - \Lambda) + S_{matter}[g, \psi_i]$$

The effect of JT gravity on the scattering problem at the nonperturbative level is to make the coordinates dynamical,

$$\sigma^\pm \to X^\pm \equiv \frac{2}{\Lambda}\partial_\mp \phi = \sigma^\pm + Y^\pm$$

Using the EOM, the $Y^\pm$ coordinates can be shown to satisfy

$$Y^\pm(\sigma^0 = -\infty, \sigma^1_i) = \pm\frac{1}{2\Lambda}\left[\tilde{P}^\pm_>(\sigma^1_i) - \tilde{P}^\pm_<(\sigma^1_i)\right]$$

where $\tilde{P}^\pm_>(\sigma_i)$ denotes the momenta of all the particles with $\sigma^1 > \sigma^1_i$ while $\tilde{P}^\pm_<(\sigma_i)$ are the momenta of particles with $\sigma^1 < \sigma^1_i$.

In the absence of gravity, an in state is defined (up to normalization) by

$$|p\rangle_{in} = a_i^\dagger(p)|0\rangle$$

where the creation operator $a_i^\dagger(p)$ is defined at $\sigma^0 \to -\infty$ via

$$\psi_i = \int_{-\infty}^{\infty} \frac{dp}{\sqrt{2\pi}} \frac{1}{\sqrt{2E}} \left[a_i^\dagger(p) e^{ip_\mu \sigma^\mu} + h.c. \right]$$

Once we turn on gravity and swicth to the dynamical coordinates we have

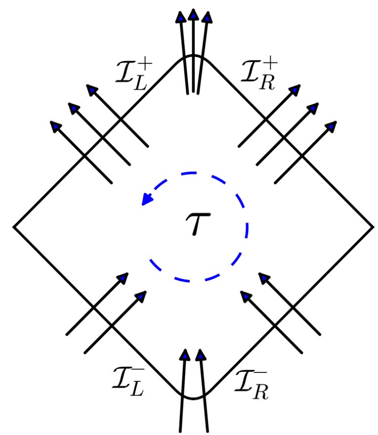
$$\psi_i = \int_{-\infty}^{\infty} \frac{dp}{\sqrt{2\pi}} \frac{1}{\sqrt{2E}} \left[\underbrace{a_i^\dagger(p) e^{ip_\mu Y_\mu}}_{\equiv A_i^\dagger(p)} e^{ip_\mu \sigma^\mu} + h.c. \right]$$

where $A_i^\dagger(p) \equiv a_i^\dagger(p) e^{ip_\mu Y^\mu} = a_i^\dagger(p) e^{-i(p^+ Y^- + p^- Y^+)}$ is the gravitationally-drossed creation operator.

Thus, for a general in state we have:

$$\underbrace{\widetilde{|\{p_i\}\rangle}_{in}}_{\text{dressed in-state}} = \prod_i A^\dagger(p_i)\,|0\rangle = e^{-\frac{i}{2\Lambda}\underbrace{\sum_{i<j} p_i * p_j}_{\substack{\text{ordered according to}\\ \text{the rapidities } \beta_i}}}\,|\{p_i\}\rangle_{in}\,, \qquad p_i * p_j \equiv \epsilon_{\mu\nu}\, p_i^\mu\, p_j^\nu$$

$\longrightarrow$ compatible with the previous assumption



Similarly for the out states we have

$$\widetilde{|\{q_i\}\rangle}_{out} = \prod_i A^\dagger(q_i)\,|0\rangle = e^{\frac{i}{2\Lambda}\sum_{i<j} q_i * q_j}$$

Altogether the dressed S-matrix is given by

$$\tilde{S} = {}_{out}\langle \widetilde{\{q_i\}} | \widetilde{\{p_i\}} \rangle_{in} = {}_{out}\langle \{q_i\} | \{p_i\} \rangle \, e^{-\frac{i}{2\Lambda} \sum_{i<j} p_i * p_j - \frac{i}{2\Lambda} \sum_{i<j} q_i * q_j}$$

$$\tilde{S} = \underset{\text{original S-matrix}}{S} \, \underbrace{e^{-\frac{i}{2\Lambda} \sum_{i<j} p_i * p_j - \frac{i}{2\Lambda} \sum_{i<j} q_i * q_j}}_{\text{generalization of the CDD factor}}$$

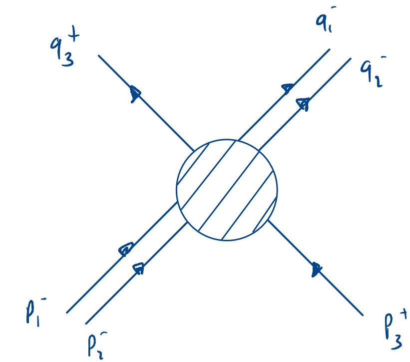
<u>Comments</u>:

- The dressed S-matrix satisfies the physical conditions described in previous lectures, namely it's unitary, analytic, and crossing symmetric (provided the bare S-matrix also satisfies these properties).
- $\tilde{S}$ is the S-matrix of general $T\bar{T}$-deformed QFTs provided we identify

$$\mu = 2\Lambda^{-1}$$

- As a consistency check let us consider the scattering of 3 particles when the undeformed theory is a free scalar.

The gravitational dressing factor for the in states reads



$$U = \exp\left\{ -\frac{i}{2\Lambda}\left(p_1^+ \underset{\substack{\downarrow \\ -p_2^- - p_3^-}}{y_1^-} + p_1^- \underset{\substack{\downarrow \\ p_2^+ + p_3^+}}{y_1^+} + p_2^+ y_2^- + p_2^- y_2^+ + p_3^+ y_3^- + p_3^- y_3^+ \right)\right\}$$

$$= \exp\left\{ \frac{i}{\Lambda} \underbrace{\left(-p_1^- p_2^+ + p_1^+ p_2^- - p_1^- p_3^+ + p_1^+ p_3^- - p_2^- p_3^+ + p_2^+ p_3^- \right)}_{\sum_{i<j} p_i * p_j} \right\}$$

The momenta satisfy $p_1^- = q_1^-$, $p_2^- = q_2^-$, $p_3^+ = q_3^+$, so we can write

$$U(p_i) = e^{\frac{i}{4\Lambda}\left(s_{12} + s_{13} + s_{23}\right)} \qquad \text{where} \qquad s_{ij} = -(p_i + p_j)^2$$

$$U(q_i) = U(p_i)$$

Finally letting $\mu = 2\Lambda^{-1}$ we recover the S-matrix of a $T\bar{T}$-deformed free scalar

$$\tilde{S} = U(q_i)\, S\, U(p_i) = \mathbb{1}\, e^{i\mu(s_{12} + s_{13} + s_{23})/4}$$

Perturbative deformation

The derivation of the S-matrix above is non perturbative and holds to all orders in Λ^{-1}.

Perturbatively we can also show that coupling a QFT to flat JT gravity is equivalent to adding the $T\bar{T}$ operator to the classical action.

In order to see this we work with the original (non dynamical) coordinates $\sigma^{\pm}$ – the analog of the string in the static gauge – and expand the fields to linear order around the vacuum

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad \phi = -\frac{\Lambda}{2}\sigma^+\sigma^- + c + \varphi$$

The quadratic action is then given by

$$S^{(2)}_{JT} = \int \varphi\left(\partial_+^2 h_{--} + \partial_-^2 h_{++} - 2\partial_+\partial_- h_{+-}\right) + \frac{\Lambda}{4}\left(h_{++}h_{--} - 2h_{+-}^2\right) + \frac{1}{2}h_{++}T_{--} + \frac{1}{2}h_{--}T_{++} + h_{+-}T_{+-}$$

$$+ \frac{\Lambda}{4}\left[\sigma^+ h_{++}\left(2\partial_- h_{+-} - \partial_+ h_{--}\right) + \sigma^- h_{--}\left(2\partial_+ h_{+-} - \partial_- h_{++}\right)\right]$$

and the EOM read

$$\partial_+^2 h_{--} + \partial_-^2 h_{++} - 2\partial_+\partial_- h_{+-} = 0$$

$$\partial_\pm^2 \varphi + \frac{\Lambda}{4}\left[(2 \pm \sigma^+\partial_+ \mp \sigma^-\partial_-)\, h_{\pm\pm} + 2\sigma^\mp \partial_\pm h_{+-} \right] = -\frac{1}{2} T_{\pm\pm}$$

$$\partial_+\partial_- \varphi + \frac{\Lambda}{4}\left(2h_{+-} + \sigma^+\partial_- h_{++} - \sigma^-\partial_+ h_{--} \right) = 0$$

Using conservation of the stress tensor it is not difficult to show that the first equation is solved by

$$h_{\mu\nu} = -\frac{2}{\Lambda}\left(T_{\mu\nu} - \eta_{\mu\nu} T^\alpha{}_\alpha \right)$$

whereupon the other equations become

$$\partial_\pm^2 \varphi = \frac{1}{2}(1 + \sigma^\alpha\partial_\alpha)\, T_{\pm\pm}, \qquad \partial_+\partial_-\varphi = -\frac{1}{2}(1+\sigma^\alpha\partial_\alpha)\, T_{+-}$$

$$\Downarrow$$

$$\partial_\pm \varphi = \frac{1}{2}\left(\sigma^\pm T_{\pm\pm} - \sigma^\mp T_{+-} \right)$$

Finally, plugging this solution back into the quadratic action we obtain

$$S_{JT}^{(2)} = -\frac{1}{2\Lambda}\int d^2\sigma \left(T_{\mu\nu}T^{\mu\nu} - T_{\mu}^{\mu\,2} \right) = -\mu\int d^2\sigma \left(T_{\mu\nu}T^{\mu\nu} - T_{\mu}^{\mu\,2} \right) = S_{T\bar{T}}$$

which is the leading term in the $T\bar{T}$ deformation of the QFT coupled to JT gravity.