Aspects of holography and irrelevant deformations

Lecture 12:

$T\bar{T}$ as a noncritical string

References:

1910.13578

16.06.2021

We have seen that the $T\bar{T}$ deformation of any relativistic two-dimensional QFT is equivalent to coupling the theory to flat space Jackiw - Teitelboim gravity

$$S = \int_\Sigma d^2\sigma \sqrt{|g|}\,(\phi R - \Lambda) + S_{QFT}[g, \psi_i]$$

The effect of JT gravity can be understood in terms of dynamical coordinates which, for a flat metric, were given by

$$X^\pm(\sigma) = \frac{2}{\Lambda}\,\partial_\mp \phi .$$

For more general backgrounds we turned to the vielbein formalism where the gauge fixed action becomes

$$S_{JT} = \int d^2\sigma\ \epsilon^{\alpha\beta}\left(\lambda^a \partial_\alpha e^a_\beta + \frac{\Lambda}{2}\epsilon_{ab}\, e^a_\alpha e^b_\beta\right)$$

and the dynamical coordinates read

$$\tilde{X}^a = \Lambda^{-1}\,\epsilon^a{}_b\,\lambda^b .$$

When the QFT is conformal, i.e. a CFT, there is an alternative nonperturbative description of $T\bar{T}$-deformed CFTs as a noncritical string that is equivalent to flat JT gravity on shell.

In this formulation, the derivation of the spectrum and the torus partition function are straightforward.

Let us first consider a CFT with central charge $c=24$. The $T\bar{T}$ deformation of the CFT is given by a $d=2$ critical string

$$S_{T\bar{T}} = \frac{1}{2\pi\mu} \int d^2z \; \bar{\partial} X^+ \partial X^- + S_{CFT}$$

worldsheet coordinates

target space coordinates

think of this as an internal CFT.

We interpret this action as the gauge fixed action obtained from a covariant action with a dynamical 2d worldsheet metric in the conformal gauge.

On-shell the target space coordinates satisfy

$$X^{\pm}(\sigma^+,\sigma^-) = \tilde{X}^{\pm}(\sigma^{\pm}) + Y^{\mp}(\sigma^{\mp}).$$

The target space coordinates are shift invariant and are identified with dynamical coordinates. In order to see this we use the Virasoro constraints

$$-\partial X^+ \partial X^- + T_{CFT} = 0, \qquad -\bar{\partial} X^+ \bar{\partial} X^- + \bar{T}_{CFT} = 0.$$

(here and onwards we set $\mu = 1$)

We can fix the residual conformal invariance by choosing $(z,\bar{z}) \to (\sigma^+,\sigma^-)$ with

$$\underbrace{\partial_+ X^+ = \partial_- X^- = 1}_{X^{\pm} = \sigma^{\pm} + Y^{\pm}(\sigma^{\mp})}$$

Then the $Y^{\pm}$ terms are determined by

$$\partial_{\pm} X^{\mp} + T^{CFT}_{\pm\pm} = 0 \quad \Rightarrow \quad X^{\mp} = \sigma^{\mp} + \int^{\sigma^{\pm}} d\sigma^{\pm}\, T^{CFT}_{\pm\pm}$$

We see that these coordinates are dynamical, i.e. determined by the CFT, and that they agree with the expressions for $T\bar{T}$-deformed CFTs.