Aspects of holography and irrelevant deformations

Lecture 2 :

a simple toy model of quantum gravity

References:

1703.1054, 1205.6805, Zamolodchikov '91

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Last time: infinitely long free bosonic string

$\downarrow$

shares many features of quantum gravity:

- minimal length (nonlocality).
- universal time delay ($\Delta t \propto E$)

from the S-matrix

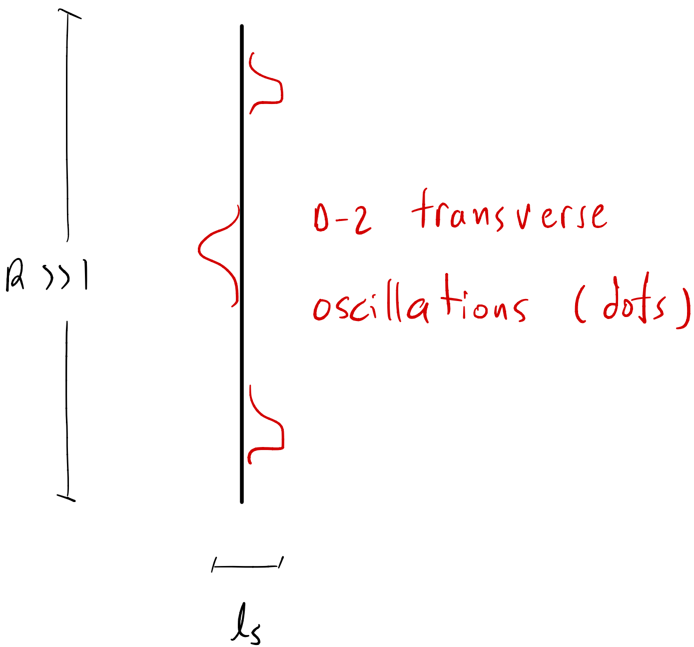
- no off-shell observables *

the theory is nonrenormalizable

- but features a "good" S-matrix in the UV.
- new kind of RG flow: asymptotic fragility

The infinitely long string

Consider an infinitely long straight string in D-dimensional Minkowski spacetime. How do we describe the transverse oscillations of the string?



Standard quantization of the string:

- Lightcone: $D=26$
- Covariant: $D-1$ dofs if $D \neq 26$
- Noncritical: $D-2+1$ dofs
  - ↳ Liouville mode

Effective field theory approach (EFT)

The string spontoneously breaks the D-dimensional Poincaré group to

$$ISO(1,D-1) \longrightarrow \underbrace{ISO(1,1)}_{\text{worldsheet Poincaré}} \otimes \underbrace{SO(D-2)}_{\text{flavor symmetry}}$$

Spontaneously broken symmetries: D-2 translations + D-2 rotations + D-2 boosts.

Goldstone's theorem:

$$D-2 \text{ fields with } \overbrace{\text{derivative interactions}}^{\text{nonlinearly realize broken Poincaré}}$$

Let $X^\mu = (\sigma, \tau, X^i(\sigma,\tau))$. Then the effective action is (CCWZ '69)

$$S = \underbrace{-\int d^2\sigma \sqrt{-h}\,\{ \ell_s^{-2}}_{\substack{\text{Nambu-Goto action}\\ \text{(in static gauge)}}} + \underbrace{\alpha_0^{-1} (K^i_{\alpha\beta})^2 + \beta_0^{-1} R + \dots \}}_{\substack{\text{all possible local invariants built out of}\\ h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu}}$$

The NG action is the universal IR description of the long string; we treat it as a nonrenormalizable 2d theory with derivative interactions

$$S_{NG} = -\frac{1}{4\ell_s^2}\int d^2\sigma \left[\# \partial_\alpha X^i \partial^\alpha X^i - \frac{1}{2} \overbrace{(\partial_\alpha X^i \partial^\alpha X^i)^2 + (\partial_\alpha X^i \partial_\beta X^i \partial^\alpha X^j \partial^\beta X^j) + \dots}^{} \right]$$

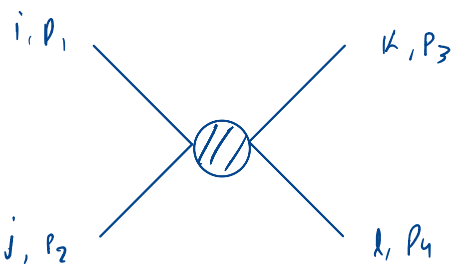
But we will see that in $D=26$ the theory is UV complete by itself.

Summary of perturbative results:

The $SO(D-2)$ flavor symmetry restricts the amplitude of $2 \to 2$ scattering:

$$M_{ij,kl} = A\, \delta_{ij,kl} + B\, \delta_{ik,jl} + C\, \delta_{il,jk}$$

annihilations



$$s = -(p_1+p_2)^2$$

$$t = -(p_1+p_3)^2$$

$$u = -(p_1+p_4)^2$$

In 2d:

$t = 0 \Rightarrow u = -s$

or

$u = 0 \Rightarrow t = -s$

We focus on the annihilation part of the amplitude (B, C from crossing).

$$A(s,u,t) = A(s,t,u) = B(t,s,u) = C(u,t,s)$$

Tree level: the results are independent of D

$$A_{tree} = -\frac{l_s^2}{2}\, tu \qquad (=0 \text{ in } 2d)$$

$$\mathcal{M}^{ij,kl}_{tree} = -\frac{l_s^2}{2}\left(\delta^{ik}\delta^{jl}\, su + \delta^{il}\delta^{jk}\, st\right).$$

One loop:

vanishes exactly in 2d $(A=B=C=0)$

$$A_{1-loop} = -\frac{l_s^4}{192\pi}\left\{ (D-26)\, s^3 + stu\left[\frac{16D+4}{3} - 2(D-8)\log\left(-\frac{s}{\mu}\right)\right] + 12\, tu\left(t\log\frac{s}{t} + u\log\frac{s}{u}\right)\right\}$$

Polchinski-Strominger term:

vanishes in 2d but $B\neq 0$, $C\neq 0$

$$S_{PS} = -\frac{(D-26)}{192\pi^2}\int d^2\sigma\, \sqrt{h}\, R\,\frac{1}{\Box}\,R$$

Lessons

1. From EFT approach $D=26$ is special since it guarantees the absence of annihilations, $A = 0 + O(\ell_s^6)$. ($D=3$ is also special!)

This is a hint of integrability of the NG theory.

2. For $D \neq 26$ we can add the PS term to the action such that

$A = 0 + O(\ell_s^6)$,

$$S_{LC} = S_{NG} + S_{PS} + O(\ell_s^2)$$

We'll come back to this later!

Note that S_{PS} breaks target space Lorentz invariance but is local in terms of the X^i fields, namely $\sqrt{-h}\, R \frac{1}{\Box} R \propto (\partial_\alpha \partial_\beta X^i \partial_\gamma X^i)^2$.

3. The log terms actually disappear and the amplitude is purely polynomial.

For example, to order $\mathcal{O}(l_s^4)$ we find that the B-term is

$$B = \frac{l_s^2}{2} s^2 + \frac{i l_s^4}{16} s^3 + \mathcal{O}(l_s^6).$$

This is another hint of the special features of the NG S-matrix.

<u>In this lecture</u>:

1. Describe general properties of the S-matrix

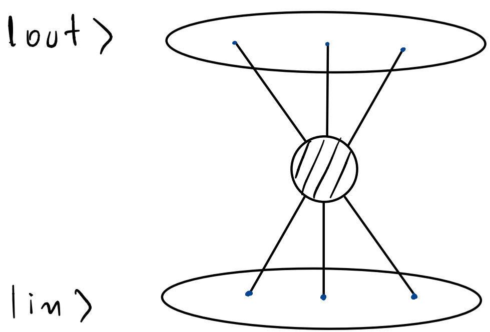
2. Generalize the critical NG string.

The S-matrix for massless fields in 2d

S-matrix: scattering of states that do not interact in the far past / future.

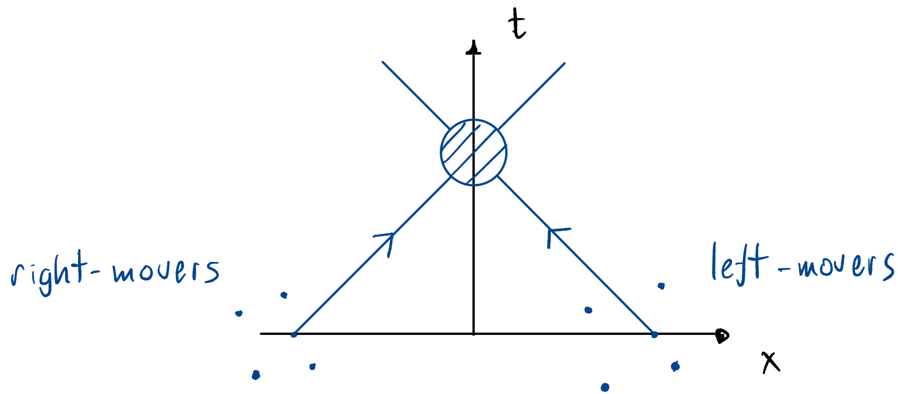
subtle in 2d

In $d>2$: it's always possible to prepare states that do not interact at ∞

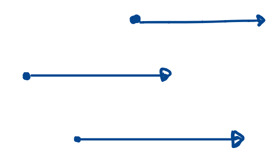


$\longrightarrow$ in $d>2$ spatial infinity is at least two-dimensional and connected

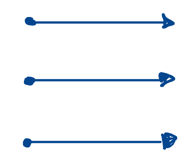
In $d=2$: spatial infinity consists of 2 disconnected points



- when $m>0$ particles are segregated according to their velocities; can define asympt. states



- when $m=0$ particles sit on top of each other; leads to IR divergences in the S-matrix



We can have a finite s-matrix in 2d if the left (right) movers do not interact with themselves and $\underbrace{1 \to \text{many splittings}}_{\text{absent perturbatively in the NG string}}$ are not possible.

(see 9310098).

absent perturbatively in the NG string

also note absence of log terms.

<u>Zamolodchikov's argument</u>: Lorentz invariance + shift symmety = no interactions among left or right-movers by themselves.

For the NG string, the perturbative analysis suggests the absence of particle production and annihilations such that the s-matrix is diagonal: a string oscillating in one direction will keep oscillating forever.

Finite volume spectrum :

In order to show that the absence of particle production and annihilations persists to all orders, i.e. nonperturbatively, we look at the spectrum.

For the critical NG string we can get the S-matrix from the finite volume spectrum. Recall the equivalence between Nambu-Goto and Polyakov

$$\underbrace{S_{NG} = -\frac{1}{\ell_s^2}\int d\tau\, d\sigma\, \sqrt{-h}}_{\text{quantization ?}} \quad \Longleftrightarrow \quad \underbrace{\begin{aligned} S_P &= -\frac{1}{\ell_s^2}\int d\tilde{\tau}\, d\tilde{\sigma}\, \sqrt{-\gamma}\, \gamma^{\alpha\beta}\, \partial_\alpha X^\mu \partial_\beta X_\mu \\ \frac{\delta S_P}{\delta \gamma^{\alpha\beta}} &= T_{\alpha\beta} = 0 \end{aligned}}_{\text{lightcone, covariant quantization}}$$

Let us consider a winding string

$$X'(\sigma + 2\pi) = X'(\sigma) + wR$$

winding $w = 1$ $R \to \infty$ for the long string

In light cone quantization, the target space energy of a winding string is

$$E_{LC} = \sqrt{\underbrace{\frac{4\pi^2 (N - \tilde{N})^2}{R^2}}_{\text{momentum}} + \underbrace{\frac{R^2}{l_s^4}}_{\text{potential energy}} + \frac{4\pi}{l_s^2}\left(\underbrace{N + \tilde{N}}_{\text{transverse oscillators}} - \underbrace{\frac{D-2}{12}}_{\text{ground state}}\right)}$$

where $N, \tilde{N}$ are the levels of the string

$$N = \sum_{i=2}^{D-2} \sum_{n=1}^{\infty} n \underbrace{a_n^{i\dagger} a_n^i} \qquad \tilde{N} = \sum_{i=2}^{D-2} \sum_{n=1}^{\infty} \underbrace{\tilde{a}_n^{i\dagger} \tilde{a}_n^i}$$

creation / annihilation operators of x^i

Note that E_{LC} is not the energy (Hamiltonian) of the Polyakov action

$$H_{LC} \sim \frac{\partial}{\partial \tilde{\tau}} \qquad\qquad E_{LC} \sim \frac{\partial}{\partial X^0(\sigma,\tau)}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$H_{LC} \sim N + \tilde{N} \qquad\qquad E_{LC} = \sqrt{\ldots}$$

But in static gauge (at least clasically)

$$E_{NG}\Big|_{D=3,\,26} = \frac{\partial}{\partial \tau} = \frac{\partial}{\partial X^0} = E_{LC}$$

$\downarrow$

<u>Why</u>? If $D \neq 3, 26$ the string in lightcone quantization is not Lorentz invariant (in the target space) whereas the NG string is by construction invariant under non linearly realized Lorentz boosts.

Definition :

The "simplest" toy model of quantum gravity is the theory whose spectrum at the quantum level is given by E_{LC}.

Comments :

- When $D = 3, 26$, this theory is the NG string in static gauge, and we assume $E_{NG} = E_{LC}$ at the quantum level (check on the IR).
- Otherwise, this is a theory with $D-2$ dofs in the IR that lacks nonlinearly realized Poincaré, i.e. it's not the NG string.
- For any D, the theory with E_{LC} is a $T\bar{T}$ - deformed CFT.