Single-trace $T\bar{T}$ deformations and string theory

Lecture I

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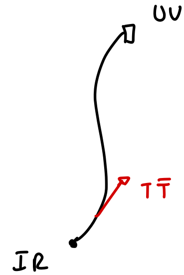
Introduction

The $T\bar{T}$ deformation of a Poincaré QFT is defined by

$$\frac{\partial S}{\partial \mu} = -4 \int dx^+ dx^- \, T\bar{T}$$



- $T\bar{T} = T_{++} T_{--} - T_{+-}^2$, $\quad x^{\pm} = t \pm x$, $\quad ds^2 = -dx^+ dx^-$
- $T_{\mu\nu} \to$ instantaneous stress energy tensor

$$S = S_0 - 4\mu \int T\bar{T}^{(0)} + \mathcal{O}(\mu^2)$$

- $[\mu] = L^2 \to$ deformation is irrelevant

Remarkable properties:

- universal $\to$ doesn't depend on details of the QFT
- solvable $\to$ spectrum + S-matrix
- nonlocal but UV complete
- not an RG flow $\to$ preserves # of dofs
- equivalent to coupling to gravity

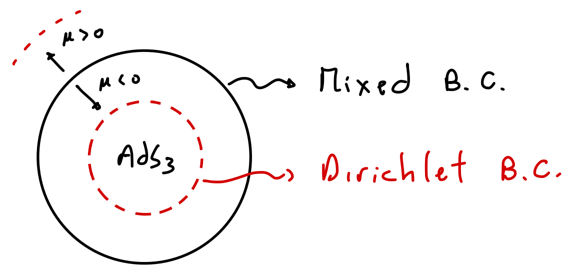
What is the holographic dual of a $T\bar{T}$-deformed CFT ?

$$\begin{tikzcd} \text{UV}: & T\bar{T} \arrow[d] \arrow[r, leftrightarrow] & ? \\ \text{IR}: & CFT_2 \arrow[r, leftrightarrow] & AdS_3 \arrow[u] \end{tikzcd}$$

Answer depends on double vs single trace version of $T\bar{T}$.

<u>Double trace</u> :

- doesn't change local geometry
- cutoff / glue-on AdS_3 or AdS_3 with mixed BC



<u>Single trace</u> :

- changes the local geometry (and B.C.) !

$$AdS_3 \longrightarrow \text{linear dilaton or TsT spacetimes}$$

- string theory realization
- toy model of non-AdS holography

Outline

1. The $T\bar{T}$ deformation
   - spectrum
   - modular invariance
   - single vs double trace

2. String theory in AdS_3
   - long string spectrum
   - holographic dual

3. $T\bar{T}$ in string theory
   - $T\bar{T}$ on the worldsheet
   - TsT black holes
   - deformed long string spectrum
   - further evidence

The $T\bar{T}$ deformation

What's so special about the $T\bar{T}$ operator?

(1) $T\bar{T} \equiv \lim_{z \to z'} \left[T_{zz}(z)\, T_{\bar{z}\bar{z}}(z') - T_{z\bar{z}}(z)\, T_{z\bar{z}}(z') \right]$ is well defined

(2) $\langle T\bar{T} \rangle = \langle T_{zz} \rangle \langle T_{\bar{z}\bar{z}} \rangle - \langle T_{z\bar{z}} \rangle^2 = \text{constant}$

$$\boxed{z \equiv x + it}$$

Assumptions:

(1) Poincaré invariance

$$\partial_\mu T^\mu{}_\nu = 0, \qquad T_{\mu\nu} = T_{\nu\mu}$$

$$\begin{aligned} \partial T_{z\bar{z}} + \bar{\partial} T_{zz} &= 0 \\ \bar{\partial} T_{z\bar{z}} + \partial T_{\bar{z}\bar{z}} &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} \bar{\partial} T &= \partial \Theta \\ \partial \bar{T} &= \bar{\partial} \Theta \end{aligned} \qquad \begin{aligned} T &\equiv T_{++} \\ \bar{T} &\equiv T_{--} \\ \Theta &\equiv T_{+-} \end{aligned}$$

(2) Global translations

$$\langle \mathcal{O}(z) \rangle = \langle 0 | U^\dagger_{\Delta z}\, \mathcal{O}(z_0)\, U_{\Delta z} | 0 \rangle = \langle \mathcal{O}(z_0) \rangle \Rightarrow \text{constant}$$

$$\langle \mathcal{O}(z)\, \mathcal{O}(z') \rangle = \langle \mathcal{O}(z_0)\, \mathcal{O}(z' - z + z_0) \rangle = f(z - z')$$

(3) Local interactions

$$\lim_{\alpha \to \infty} \langle \mathcal{O}(z + \alpha \cdot \omega)\, \mathcal{O}(z') \rangle = \langle \mathcal{O}(z) \rangle \langle \mathcal{O}(z') \rangle$$

Exercise: check for $\mathcal{O} = T$ in CFT on the cylinder.

$\Rightarrow$ theory is defined on the plane or cylinder

I. The $T\bar{T}$ operator is free of divergences

$$T\bar{T}(z) = \lim_{z\to z'} \left[T(z)\,\bar{T}(z') - \Theta(z)\,\Theta(z') \right]$$

<u>OPEs</u>:

$$\begin{pmatrix} T(z)\,\bar{T}(z') & T(z)\,\Theta(z') \\ \Theta(z)\,\bar{T}(z') & \Theta(z)\,\Theta(z') \end{pmatrix} = \sum_i \begin{pmatrix} D_i(z-z') & A_i(z-z') \\ B_i(z-z') & C_i(z-z') \end{pmatrix} \mathcal{O}_i(z')$$

$$\Rightarrow\ T(z)\bar{T}(z') - \Theta(z)\,\Theta(z') = \sum_i \left[D_i(z-z') - C_i(z-z') \right] \mathcal{O}_i(z')$$

$$= \mathcal{O}_{T\bar{T}}(z') + \tilde{\sum_i}\ \underbrace{\tilde{F}_i(z-z')}\ \mathcal{O}_i + \sum_i \underbrace{F_i(z-z')}\ \partial\mathcal{O}_i$$

potentially divergent

Using $\partial_\mu T^\mu{}_\nu = 0$ $\quad (\bar{\partial} T = \partial\Theta,\ \partial\bar{T} = \bar{\partial}\Theta)$:

- $\partial_{\bar{z}} \left[T(z)\,\bar{T}(z') - \Theta(z)\,\Theta(z') \right]$

$$= \underbrace{\partial_{\bar{z}} T(z)}_{\partial_z \Theta(z)}\ \bar{T}(z') - \partial_{\bar{z}}\Theta(z)\,\Theta(z') + \Theta(z)\underbrace{\left[\partial_{z'}\bar{T}(z') - \partial_{\bar{z}'}\Theta(z') \right]}_{0}$$

$$= \underbrace{(\partial_z + \partial_{z'})}\ \Theta(z)\,\bar{T}(z') - \underbrace{(\partial_{\bar{z}} + \partial_{\bar{z}'})}\ \Theta(z)\,\Theta(z')$$

$(\partial_z + \partial_{z'})(z-z') = 0 \qquad \ldots$

$$\Rightarrow\ \partial_{\bar{z}} \left[T(z)\,\bar{T}(z') - \Theta(z)\,\Theta(z') \right] = \sum_i \left[B_i(z-z')\,\underline{\partial_{z'}\mathcal{O}_i(z')} - C_i(z-z')\,\underline{\partial_{\bar{z}'}\mathcal{O}_i(z')} \right]$$

$$\partial_z \left[T(z)\,\bar{T}(z') - \Theta(z)\,\Theta(z') \right] = \sum_i \left[D_i(z-z')\,\underline{\partial_{z'}\mathcal{O}_i(z')} - A_i(z-z')\,\underline{\partial_{\bar{z}'}\mathcal{O}_i(z')} \right]$$

$$\boxed{\langle T(z)\,\bar{T}(z') - \Theta(z)\,\Theta(z') \rangle = \langle \mathcal{O}_{T\bar{T}}(z) \rangle + \sum_i \tilde{F}_i(z-z')\,\cancelto{0}{\langle \partial\mathcal{O}_i \rangle}}$$

Thus, the $T\bar{T}$ operator is well defined up to total derivatives.

(2) The $T\bar{T}$ operator factorizes

$$\langle T\bar{T}\rangle = \langle T\rangle\langle\bar{T}\rangle - \langle\Theta\rangle^2 = \text{constant}$$

$$\partial_{\bar{z}}\underbrace{\left(\langle T(z)\bar{T}(z')\rangle - \langle\Theta(z)\Theta(z')\rangle\right)}_{\equiv I(z,z')} = \partial_z\langle\Theta(z)\bar{T}(z')\rangle - \partial_{\bar{z}}\langle\Theta(z)\Theta(z')\rangle$$

$$= -\partial_{z'}\langle\Theta(z)\bar{T}(z')\rangle + \partial_{\bar{z}'}\langle\Theta(z)\Theta(z')\rangle$$

$$= -\langle\Theta(z)\underbrace{\left[\partial_{z'}\bar{T}(z') - \partial_{\bar{z}'}\Theta(z')\right]}_{0}\rangle$$

$$= 0$$

Easy to show: $\partial_z I = \partial_{\bar{z}} I = \partial_{z'} I = \partial_{\bar{z}'} I = 0$.

$\downarrow$

can put z, z' at any location

$$\Rightarrow \langle T(z)\,T(z')\rangle - \langle\Theta(z)\,\Theta(z')\rangle = \langle T(z)\rangle\langle\bar{T}(z')\rangle - \langle\Theta(z)\rangle\langle\Theta(z')\rangle$$

Thus, (1) + (2) :

$$\boxed{\langle T\bar{T}(z')\rangle = \lim_{z\to z'}\left(\langle T(z)\bar{T}(z')\rangle - \langle\Theta(z)\Theta(z')\rangle\right) = \langle T(z')\rangle\langle\bar{T}(z')\rangle - \langle\Theta(z')\rangle^2}$$

<u>Comments</u> :

(1) $|0\rangle \to |n\rangle$: $\quad H|n\rangle = E_n|n\rangle, \quad P|n\rangle = P_n|n\rangle$

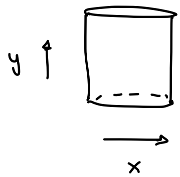
(2) We can relax Lorentz invariance, $T_{z\bar{z}} \neq T_{\bar{z}z}$

(3) Works for other conserved currents, e.g

$$J\bar{T}(z') = \lim_{z\to z'}\langle J(z)\bar{T}(z') - \bar{J}(z)\,\Theta(z')\rangle$$

The spectrum

Put the theory on a cylinder (so $\langle T\rangle \neq 0$)



$x \sim x + R$,

Consider an eigenstate $H|n\rangle = E_n|n\rangle$, $P|n\rangle = P_n|n\rangle$

$$\frac{\partial E_n}{\partial \mu} = \langle n|\frac{\partial H}{\partial \mu}|n\rangle = -4\int dx\, \langle n|T\bar{T}|n\rangle = -4R\langle n|T\bar{T}|n\rangle$$

in Euclidean signature

We need:

- $T_{zz} = \frac{1}{4}\left(T_{xx} - T_{tt} - 2iT_{xt}\right)$, $\quad T_{\bar{z}\bar{z}} = T_{zz}^{*}$, $\quad T_{z\bar{z}} = \frac{1}{4}\left(T_{xx} + T_{tt}\right)$

- $$\underbrace{\langle n|T_{tt}|n\rangle = -\frac{E_n}{R}, \quad \langle n|T_{xt}|n\rangle = \frac{iP_n}{R},}_{Q_n = i\int dx\, \langle n|T_{t\mu}|n\rangle\, \xi^\mu} \quad \langle n|T_{xx}|n\rangle = -\frac{\partial E_n}{\partial R}$$

Altogether we obtain

$$\langle n|T\bar{T}|n\rangle = \langle n|T_{zz}|n\rangle\langle n|T_{\bar{z}\bar{z}}|n\rangle - \langle n|T_{z\bar{z}}|n\rangle^2$$

$$= -\frac{1}{4R}\left(E_n\frac{\partial E_n}{\partial R} + \frac{P_n^2}{R}\right)$$

$$\Rightarrow \quad \boxed{\frac{\partial E_n}{\partial \mu} = E_n\frac{\partial E_n}{\partial R} + \frac{P_n^2}{R}}$$

$\hookrightarrow$ inviscid Burger's eq.

Solution:

- $x \sim x + R \Rightarrow P_n(0) = \frac{m}{R}, \quad m \in \mathbb{Z}$
  $\Rightarrow P_n(\mu) = \frac{m}{R}$ $\Big\}\ P_n(\mu) = P_n(0)$

- Exercise: show that Burger's eq. can be written as

$$R E_n(0) = R E_n(\mu) + \mu \left[E_n(\mu)^2 - P_n(0)^2 \right]$$

(hint: $R E_n(\mu) = f(\mu/R^2)$)

or: $R E_{L,R}(0) = R E_{L,R}(\mu) + 2\mu E_L(\mu) E_R(\mu)$, $\quad E_{L/R} = \frac{E_n \pm P_n}{2}$

$$\Rightarrow \boxed{E_n(\mu) = -\frac{R}{2\mu}\left(1 - \sqrt{1 + \frac{4\mu}{R} E_n(0) + 4\frac{\mu^2}{R^2} P_n(0)^2}\right), \quad P_n(\mu) = P_n(0)}$$

Comments:

- Universal for any Poincaré invariant QFT
- $\lim_{R\to 0} E_0(\mu) \not\propto 1/R \Rightarrow$ UV is not a CFT fixed point
- For a CFT:

$$R E_n(0) = h_n + \bar{h}_n - \frac{c}{12}, \qquad R P_n(0) = h_n - \bar{h}_n \qquad \begin{array}{l} L_0 |n\rangle = h_n |n\rangle \\ \bar{L}_0 |n\rangle = \bar{h}_n |n\rangle \end{array}$$

$$E_n(\mu) = -\frac{R}{2\mu} + \sqrt{\frac{R^2}{4\mu^2} + \frac{1}{\mu}\underbrace{\left(h_n + \bar{h}_n - \frac{c}{12}\right)}_{<0 \text{ for the CFT vacuum } h_n = \bar{h}_n = 0} + \frac{1}{R^2}(h_n - \bar{h}_n)^2}$$

- Matches the energy of a winding string ($w=1$, $p^i=0$)

$$m^2 = \frac{w^2R^2}{l_s^4} + \frac{2}{l_s^2}\left(N+\bar{N}-\frac{D-2}{12}\right) + \frac{(N-\bar{N})}{R^2} \tag{8.3.2}$$

where $\mu = \frac{l_s^2}{2}$, $c = D-2$, $(h_n, \bar{h}_n) = (N, \bar{N})$

$\Rightarrow$ $\mathcal{L} = \partial\varphi\bar{\partial}\varphi \xrightarrow[T\bar{T}]{} \underbrace{\frac{1}{2\mu}\sqrt{1-4\mu\,\partial\varphi\bar{\partial}\varphi}-\frac{1}{2\mu}}$

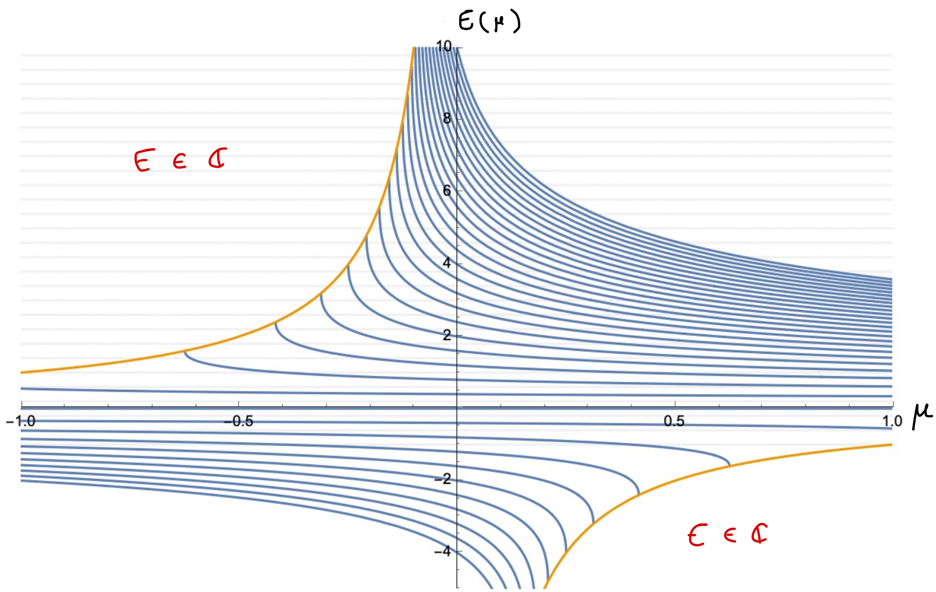
Nambu-Goto: $2\mu = l_s^2$, $x^0=\tau$, $x^1=\sigma$, $x^2=l_s\varphi$

- $\mu>0$: $\mathcal{E}_n(\mu)\in\mathbb{R}$ if $\mu \lesssim -\frac{R}{4\mathcal{E}_0(0)} = \frac{3R^2}{c}$

  $\hookrightarrow$ $\exists$ of a maximum (Hagedorn) temperature

- $\mu<0$: $\mathcal{E}_n(\mu)\in\mathbb{C}$ when $\mathcal{E}_n(0) > \frac{R}{4|\mu|} + \frac{|\mu|}{R}P_n(0)^2$

  $\hookrightarrow$ geometrical counterpart in the holographic dual



Modular invariance

Consider a CFT on a torus $z \sim z + R \sim z + \tau$. $\qquad R = 1$

$$Z_0(\tau, \bar\tau) = \mathrm{Tr}\left(q^{h_n - c/24} \bar q^{\bar h_n - c/24} \right), \qquad q = e^{2\pi i \tau}$$

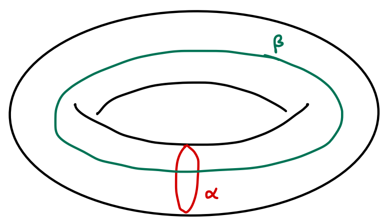
$$= \mathrm{Tr}\left(e^{-\beta E_n + i\Omega P_n} \right), \qquad \tau = \frac{\Omega + i\beta}{2\pi}$$

Invariant under large diffs $Z_0(\gamma\tau, \gamma\bar\tau) = Z_0(\tau, \bar\tau)$ where

$$\tau \to \tau = \frac{a\tau + b}{c\tau + d}, \qquad ad - bc = 1.$$

T: $\tau \to \tau + 1$, Dehn twist along α

$\Rightarrow h_n - \bar h_n \in \mathbb{Z}$



S: $\tau \to -\frac{1}{\tau}$, $\quad \alpha \leftrightarrow \beta$

$\Rightarrow$ Cardy's formula, $\rho \sim e^S$

$$S = 2\pi \left(\sqrt{\tfrac{c}{6}\left(h_n - \tfrac{c}{24}\right)} + \sqrt{\tfrac{c}{6}\left(\bar h_n - \tfrac{c}{24}\right)} \right), \qquad h_n, \bar h_n \gg c \text{ or } c \to \infty + \ldots$$

$\downarrow$

matches entropy of BTZ black holes

$\Rightarrow$ modular bootstrap $\to$ constraints on the spectrum

For $T\bar T$-deformed CFTs:

$$Z(\gamma\tau, \gamma\bar\tau; \gamma\hat\mu) = Z(\tau, \bar\tau; \hat\mu), \qquad \hat\mu \equiv \frac{\mu}{R^2}, \qquad \gamma\hat\mu = \frac{\hat\mu}{|c\tau + d|^2}$$

$\downarrow$

not related to symmetries of $T\bar T$ in an obvious way.

Perturbative proof:

$$Z(\tau,\bar\tau;\hat\mu) = \mathrm{Tr}\left(e^{-2\pi\tau_2 E_n(\hat\mu) + 2\pi i \tau_1 P_n}\right), \qquad \tau = \tau_1 + i\tau_2$$

$$E_n(\hat\mu) = \underbrace{E_n}_{\equiv E_n(0)} - (E_n^2 - P_n^2)\,\hat\mu + \mathcal{O}(\hat\mu^2)$$

$$\equiv \sum_{k=0}^{\infty} Z_k(\tau,\bar\tau)\,\hat\mu^k$$

where $Z_k(\tau,\bar\tau) \equiv \mathrm{Tr}\left(f_n^{(k)}(E_n,P_n)\, e^{-2\pi\tau_2 E_n + 2\pi i \tau_1 P_n}\right) = D^{(k)} Z_0(\tau,\bar\tau)$

CFT partition function

e.g. $Z_1(\tau,\bar\tau)$: $\quad f_n^{(1)} = 2\pi\tau_2(E_n^2 - P_n^2), \qquad D^{(1)} = \frac{2}{\pi}\tau_2\,\partial_\tau\partial_{\bar\tau}$

$Z_2(\tau,\bar\tau)$: $\quad f_n^{(2)} = 4\pi^2\tau_2^2(E_n^2 - P_n^2)^2 - 2\pi\tau_2 E_n(E_n^2 - P_n^2)$

$$D^{(2)} = \frac{2}{\pi^2}\tau_2^2\,\partial_\tau^2\partial_{\bar\tau}^2 + \frac{4i}{\pi^2}\tau_2(\partial_\tau - \partial_{\bar\tau})\,\partial_\tau\partial_{\bar\tau}$$

$\vdots$

Exercise: show that under modular transformations $\tau \to \gamma\tau$

$D^{(1)} \to (c\tau+d)(c\bar\tau+d)\,D^{(1)}$ (modular form of weight 1)

More generally one finds that for small values of $k = 1, 2, 3, \ldots$

$$D^{(k)} \to (c\tau+d)^k (c\bar\tau+d)^k D^{(k)} \implies Z_k(\gamma\tau,\gamma\bar\tau) = (c\tau+d)^k(c\bar\tau+d)^k Z_k(\tau,\bar\tau)$$

$\implies$ $\boxed{Z(\tau,\bar\tau;\hat\mu) \text{ is modular invariant if } \gamma\hat\mu = \frac{\hat\mu}{(c\tau+d)(c\bar\tau+d)}}$

We now show this works for all k.

Burger's eq. $\Rightarrow$ differential eq. for the partition function

$$-\pi^2 \tau_2 \, T_r \Big[\underbrace{\partial_{\hat\mu} E_n(\hat\mu) + 2\hat\mu\, E_n(\hat\mu)\, \partial_{\hat\mu} E_n(\hat\mu) + E_n(\hat\mu)^2 - P_n^2}_{0} \Big] e^{\cdots} = 0$$

here $E_n \to R E_n$, $P_n \to R P_n$

$$\Rightarrow \Big[\frac{\pi}{2} \partial_{\hat\mu} - \tau_2 \partial_\tau \partial_{\bar\tau} - \frac{1}{2}\Big(\partial_{\tau_2} - \frac{1}{\tau_2}\Big) \hat\mu\, \partial_{\hat\mu} \Big] Z(\tau, \bar\tau; \hat\mu) = 0$$

Using the perturbative expansion $Z(\tau,\bar\tau;\hat\mu) = \sum_{k=0}^{\infty} Z_k(\tau,\bar\tau)\, \hat\mu^k$,

$$Z_{p+1}(\tau,\bar\tau) = \frac{1}{p+1} \Big[\underbrace{\tau_2 \Big(\partial_\tau - \frac{ip}{2\tau_2}\Big)\Big(\partial_{\bar\tau} + \frac{ip}{2\tau_2}\Big)}_{(1,1)} - \underbrace{\frac{p(p+1)}{4\tau_2}}_{(1,1)} \Big] \underbrace{Z_p(\tau,\bar\tau)}_{(p,p)}$$

Thus, $Z_{p+1}(\tau,\bar\tau)$ has weight $(p+1, p+1)$. By induction,

$$\boxed{Z(\gamma\tau, \gamma\bar\tau; \gamma\hat\mu) = Z(\tau,\bar\tau;\hat\mu), \qquad \gamma\hat\mu = \frac{\hat\mu}{(c\tau+d)(c\bar\tau+d)}}$$

<u>Consequences</u>:

- Maximum temperature:

$$\frac{\mu}{R^2} \le \frac{3}{c} \xrightarrow[\tau \to -\frac{1}{\tau}]{} |\tau|^2 \ge \frac{c\mu}{3R^2}, \qquad |\tau|^2 \to \frac{\beta^2}{4\pi^2}$$

- Asymptotic density of states.

$$E_0(\hat\mu) = \frac{1}{2\hat\mu}\Big(\sqrt{1 - \frac{c\hat\mu}{3}} - 1\Big)$$

large c + sparse spectrum $\Rightarrow Z(\tau,\bar\tau;\hat\mu) \approx \begin{cases} e^{-\beta E_0(\hat\mu)}, & \beta > 2\pi \\ e^{-\beta' E_0(\hat\mu')}, & \beta < 2\pi \end{cases}$

large c: $c \to \infty$, $\hat\mu c$ fixed

sparse spectrum: light states

analog of HKS

$$S(\tau,\bar\tau) = (1 - \tau\,\partial_\tau - \bar\tau\,\partial_{\bar\tau}) \log Z(\tau,\bar\tau;\hat\mu)$$

$$= \frac{i\pi c}{6}\,\frac{1}{\gamma}\left(\frac{1}{\tau}-\frac{1}{\bar\tau}\right), \qquad \gamma = \sqrt{1-\frac{\hat\mu c}{3|\tau|^2}} \xrightarrow[\hat\mu\to 0]{} 1$$

<u>Exercise</u>: use $\hat{\mathcal{E}}_L(\hat\mu) \equiv \langle \hat{\mathcal{E}}_L(\hat\mu)\rangle_{\tau,\bar\tau} = \frac{1}{2\pi i}\,\partial_\tau \log Z(\tau,\bar\tau;\hat\mu)$ to show

$$\boxed{S(\hat\mu) = 2\pi\left(\sqrt{\frac{c}{6}\,R\,\mathcal{E}_L(\mu)\left[1+\frac{2\mu}{R}\,\mathcal{E}_R(\mu)\right]} + \sqrt{\frac{c}{6}\,R\,\mathcal{E}_R(\mu)\left[1+\frac{2\mu}{R}\,\mathcal{E}_L(\mu)\right]}\right)}$$

- Hagedorn growth at high energies: not a CFT

$$\mathcal{E}_L(\hat\mu) = \mathcal{E}_R(\hat\mu) = \frac{1}{2}\,\mathcal{E}(\hat\mu), \qquad S(\hat\mu) \approx 2\pi\sqrt{\frac{c}{3}\frac{\mu}{R^2}}\;\mathcal{E}(\hat\mu), \quad \mathcal{E}(\hat\mu) \gg 1$$

Consistency check: $\tau = i\beta/2\pi$

$$Z(\tau,\bar\tau,\hat\mu) = \mathrm{Tr}\left(e^{-\beta\mathcal{E}}\right) = \sum_{\mathcal{E}} \rho(\mathcal{E})\, e^{-\beta\mathcal{E}} = \sum_{\mathcal{E}} e^{\left(2\pi\sqrt{\frac{c}{3}\frac{\mu}{R^2}} - \beta\right)\mathcal{E}(\hat\mu)} + \dots$$

max. temperature: $\beta \geq 2\pi\sqrt{\frac{c}{3}\frac{\mu}{R^2}}$

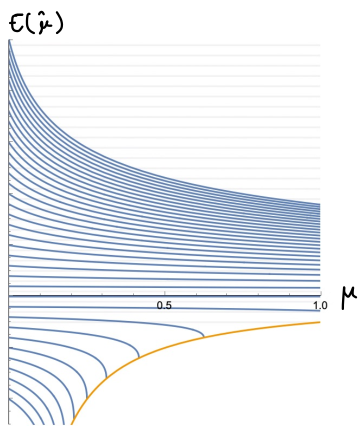
- Cardy's formula using the deformed spectrum

$$S(\hat\mu) = 2\pi\left(\sqrt{\frac{c}{6}\,R\,\mathcal{E}_L(0)} + \sqrt{\frac{c}{6}\,R\,\mathcal{E}_R(0)}\right) = S(0)$$

↳ same density of ~~high energy~~ states

$\Downarrow$

no level crossing



- $T\bar T$ is not an RG flow → it doesn't add new dots.