

Problem set 4 (due January 5)

1. Consider a particle of mass m moving in one dimension under a constant acceleration a .
 - (a) (3pts) Solve the Hamilton-Jacobi equation
 - (b) (2pt) Identify the constant α in the solution to the Hamilton-Jacobi equation with the energy. From the expression for the conjugate variable β , find the solution to the position of the particle in terms of α and β .
 - (c) (2pt) Express the previous solution in terms of the initial position and velocities of the particle at time t_0 , i.e. in terms of $x(t_0)$ and $\dot{x}(t_0)$, and verify that it takes the expected form.
2. Consider a particle of mass m subject to a time-dependent potential such that the Hamiltonian has the form

$$H = \frac{p^2}{2m} - mbtx \quad (1)$$

where b is a constant.

- (a) (3pt) Solve the Hamilton-Jacobi equation. The solution should depend on one constant which we denote by α . (There is another constant in the solution which can be removed by the same argument presented in class).
- (b) (2pts) Write the equation for the variable β conjugate to α . Using this expression, write the solution for the position of the particle as a function of α and β .
- (c) (2pt) Determine the value of α and β such that the following initial conditions are satisfied

$$x(0) = 0, \quad \dot{x}(0) = v_0 \quad (2)$$

and express the solution for the position of the particle entirely in terms of v_0 .

3. Consider the following vectors

$$\xi = (-1, 3, 2), \quad \eta = (1, 1, 1), \quad \gamma = (-2, 6, 4) \quad (3)$$

- (a) (2pts) Compute the value of the following 1-forms on $\vec{\xi}$ at the point $\vec{x} = (2, 0, 0)$

$$w_1 = \sum_{i=1}^3 dx^i, \quad w_2 = x^1 x^2 dx^3, \quad w_3 = dr^2, \quad (4)$$

where $r^2 = \sum_{i=1}^3 (x^i)^2$

- (b) (2pts) Compute the value of the following 2-forms on $\vec{\xi}$ and $\vec{\eta}$ at the point $\vec{x}$

$$w_1 = dx^2 \wedge dx^3, \quad w_2 = x^1 dx^3 \wedge dx^2, \quad w_3 = dx^3 \wedge dr^2 \quad (5)$$

- (c) (2pts) Compute the value of the following 3-form on $\vec{\xi}$, $\vec{\eta}$, and $\vec{\gamma}$ at the point $\vec{x}$

$$w = dr^2 \wedge dr^2 \wedge dr^2 \quad (6)$$

4. (Bonus, 3pts) Show that the Poisson brackets between two functions f and g on phase space can be written in terms of the symplectic structure w^2 as

$$\{f, g\} = w^2(\vec{v}_g, \vec{v}_f) \quad (7)$$

where $\vec{v}_f$ and $\vec{v}_g$ are the Hamiltonian vector fields associated with f and g , respectively.