Problem set 1 (due March 6)

1. Consider an n-dimensional space and assume a Cartesian coordinate system. Find a set of vectors $\{\vec{v}_1, \vec{v}_2, \ldots \vec{v}_n\}$ that are orthogonal to each other, meaning that $\vec{v}_i \cdot \vec{v}_j = 0$ for all $i \neq j$, and have unit norm so that $\vec{v}_i \cdot \vec{v}_i = 1$ for all i. This set of vectors forms an orthonormal basis. Show that any vector can be written in terms of this basis.

2. Consider a two-dimensional plane and a set of n vectors $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n$ where $\vec{r}_k = (\cos\,\theta_k,\, \sin\,\theta_k)$ with $\theta_k = \frac{2\pi k}{n}$ for $k = 1, 2, \ldots, n$.

   (a) Compute explicitly the sum of the vectors for $n = 2, 3, 4, 5$.

   (b) What is the sum of the vectors for general n? Provide a proof.

3. A ball is thrown straight up from the edge of the roof of a building. A second ball is dropped from the roof 1.00 s later. Ignore air resistance. Answer the following questions:

   (a) As these balls fall, does the distance between them increase, decrease, or remain the same?

   (b) If the height of the building is $20m$, what must the initial speed of the first ball be if both are to hit the ground at the same time? On the same graph, sketch the positions of both balls as a function of time, measured from when the first ball is thrown. Consider the same situation, but now let the initial speed v_0 of the first ball be given and treat the height h of the building as an unknown.

   (c) What must the height of the building be for both balls to reach the ground at the same time if (i) $v_0 = 6.0\,m/s$ and (ii) $v_0 = 9.5\,m/s$?

   (c) If $v_0 > v_{\max}$ for some value $v_{\max}$, no value of h exists that allows both balls to hit the ground at the same time. Solve for $v_{\max}$. The value $v_{\max}$ has a simple physical interpretation. What is it?

   (d) If $v_0 < v_{\min}$, no value of h exists that allows both balls to hit the ground at the same time. Solve for $v_{\min}$. The value $v_{\min}$ also has a simple physical interpretation. What is it?

4. A rocket designed to place small payloads into orbit is carried to an altitude of 10.0 km above sea level by a converted airliner. When the airliner is flying in a straight line at a constant speed of 800 km/h, the rocket is dropped. After the drop, the airliner maintains the same altitude and speed and continues to fly in a straight line. The rocket falls for a brief time, after which its rocket motor turns on. Once that motor is on, the combined effects of thrust and gravity give the rocket a constant acceleration of magnitude 3.00 g directed at an angle of 30.0° above the horizontal. For safety, the rocket should be at least 1.00 km in front of the airliner when it climbs through the airliner's altitude. Your job is to determine the minimum time that the rocket must fall before its engine starts. Ignore air resistance. Your answer should include (i) a diagram showing the flight paths of both the rocket and the airliner, labeled at several points with vectors for their velocities and accelerations; (ii) an graph of $x(t)$ showing the motions of both the rocket and the airliner; and (iii) a graph of $y(t)$ showing the motions of both the rocket and the airliner. In the diagram and the graphs, indicate when the rocket is dropped, when the rocket motor turns on, and when the rocket climbs through the altitude of the airliner.