Problem set 12 (due June 16)

1. Consider a square loop of length $4L$ with current I and magnetic dipole moment $\vec{\mu}$. The magnetic dipole moment makes an angle θ_1 with the positive z-axis.

   (a) (0.5pt) Does the angle θ_1 increase or decrease when a uniform magnetic field $\vec{B}$ pointing in the $\hat{z}$ direction is turned on?

   (b) (1.5pt) Compute the work done by the magnetic field $\vec{B}$ as the loop rotates from θ_1 to θ_2

   (c) (1.5pt) Use the expression above to find an expression for the potential energy of the loop in terms of the magnetic dipole moment.

   (d) (0.5pt) How does this expression compare to the potential energy of an electric dipole in the presence of a uniform electric field?

2. Consider a square loop on the xy-plane with current I flowing clockwise. The corners of the loop are located at $(0, 0)$, $(0, L)$, $(L, 0)$, and (L, L). There is a nonuniform magnetic field that is given by

$$\vec{B} = \frac{B_0 z}{L}\hat{y} + \frac{B_0 y}{L}\hat{z}$$

   where B_0 is a positive constant.

   (a) (1pt) Sketch the magnetic field lines in the yz-plane

   (b) (1.5pt) Find the magnitude and direction of the magnetic force exerted on each side of the loop

   (c) (0.5pt) Find the magnitude and direction of the net magnetic force on the loop

3. Consider a long straight wire of radius R along the z-axis. The wire carries a current I (directed towards the positive z-axis) that is distributed uniformly over its cross-section.

   (a) (1.5pt) Use Ampere's law to find the magnetic field at a radius r inside the wire.

   (b) (1pt) Use Ampere's law to find the magnetic field at a radius r outside the wire.

   (c) (0.5pt) Plot the magnitude of the magnetic field as a function of r