Problem set 2 (due March 13)

1. Consider a general transformation between an inertial frame O and a frame O' such that the position $\vec{x}'$ of a point particle in O' is given with respect to the position $\vec{x}$ in O by

$$\vec{x}' = f(\vec{x}, t) \tag{5}$$

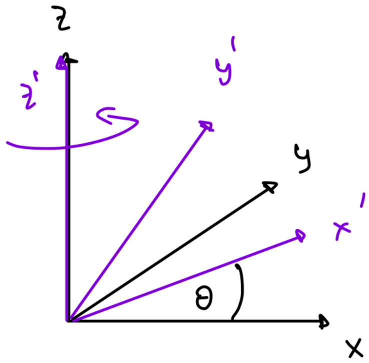
   (a) Find the conditions $f(x, t)$ must satisfy such that Newton's second law holds in O'.

   (b) How are these conditions related to the Galilean transformations?

   (c) Find the composition rule for two Galilean transformations and show that the Galilean group does indeed satisfy all the axioms of a group.

2. Consider a ball falling in some liquid. The liquid exerts a damping force $\vec{F}_d = -k\,|\vec{v}|\vec{v}$ whose magnitude is proportional to the squared speed $|\vec{v}|^2$ of the ball, and whose direction is opposite to that of the velocity. Here k is the damping constant. This force is known as quadratic drag and it holds for objects moving fast through viscous fluids. Suppose the ball is released from rest and has mass m. Determine the velocity of the ball when $t \to \infty$. This is known as the terminal velocity.

3. Consider a frame O' rotating with respect to a frame O around the z axis (see figure below). The angular velocity vector $\vec{\omega}$ always points along the axis of rotation (the z axis) and its magnitude is $|\vec{\omega}| = \dot{\theta}(t)$. Let $\vec{e}_i$ with $i = 1, 2, 3$ denote the unit vectors of frame O along the x, y, and z directions, respectively. (The unit vectors satisfy $\vec{e}_i \cdot \vec{e}_i = 1$ for all i and $\vec{e}_i \cdot \vec{e}_j = 0$ for all $i \neq j$). Similarly, let $\vec{e}_i'$ denote the unit vectors of the frame $\vec{O}'$. In particular we have $\vec{e}_3 = \vec{e}_3'$ since the z axis does not rotate.



   (a) Show that, from the point of view of O, a particle at rest in O' moves with velocity

$$\dot{\vec{r}} = \vec{\omega} \times \vec{r} \tag{6}$$

   where the cross product is given by $\vec{e}_i \times \vec{e}_j = \epsilon_{ijk}\vec{e}_k$ where ϵ_{ijk} is the Levi-Civita symbol with $\epsilon_{123} = 1$. Similarly, show that the unit vectors rotate with velocity

$$\dot{\vec{e}}_i' = \vec{\omega} \times \vec{e}_i' \tag{7}$$

(b) Denote by $\dot{\vec{r}}_O = \sum_{i=1}^{N} \dot{r}_i \vec{e}_i$ the velocity observed from frame O, and by $\dot{\vec{r}}_{O'} = \sum_{i=1}^{N} \dot{r}'_i \vec{e}'_i$ the velocity observed in frame O'. Show that

$$\dot{\vec{r}}_O = \dot{\vec{r}}_{O'} + \vec{\omega} \times \vec{r} \tag{8}$$

(c) Compute now the accelerations $\ddot{\vec{r}}_O$ and $\ddot{\vec{r}}_{O'}$. Since O' is not an inertial frame, these accelerations are not equal, but are related by the addition of extra terms. These terms are interpreted as fictitious forces that arise from working in a non-inertial frame. Show in a diagram the directions where these forces would point given some $\vec{r}$.