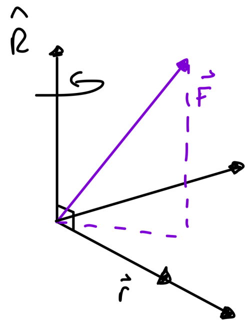
Problem set 4 (due March 27)

1. Use the Levi-Civita symbol to prove the following properties of the cross product

   (a) (1pt) $\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$

   (b) (1pt) $\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$ (note that using this identity it's easy to prove the Jacobi identity).

   (d) (0.5pt) Show that $(\vec{r}\times\vec{F})\hat{R} = \vec{r}\times\vec{F}_\perp$ where $\hat{R}$ is the unit vector along the axis of rotation and $\vec{F}_\perp$ is the component of $\vec{F}$ perpendicular to $\hat{R}$ and $\vec{r}$. You must use one of the identities above.



2. (2.5pts) A small block with mass $0.0400kg$ slides in a vertical circle of radius $R = 0.500m$ on the inside of a circular track. During one of the revolutions of the block, when the block is at the bottom of its path, point A, the normal force exerted on the block by the track has magnitude $3.95N$. In this same revolution, when the block reaches the top of its path, point B, the normal force exerted on the block has magnitude $0.680N$. How much work is done on the block by friction during the motion of the block from point A to point B?

3. (2.5pts) Consider an object of mass m in free fall in a fluid with linear drag $\vec{f} = -b\vec{v}$ (we discussed this problem in class) Compute the energy as a function of time and show that $\Delta E = E(t) - E(0) = \int_0^{y(t)} \vec{f}\cdot d\vec{y}$ for any arbitrary time t.

4. Consider a one dimensional version of the "Mexican hat" potential that is given by $U(x) = -\alpha x^2 + \beta x^4$ where $\alpha > 0$ and $\beta > 0$ (a higher dimensional version of this potential is responsible for the "Higgs mechanism" that gives particles mass)

   (a) (1pt) Find the location of the local maximum and minima of this potential

   (b) (0.5pt) If you release the particle from the local maximum (by giving it a small perturbation), what is its speed at the local minima? How far does the particle get before turning back?

   (c) (1pt) Find the period of oscillation for a particle stuck around one of the local minima assuming that $x - x_0$ is small where x_0 is the location of the local minima.