Problem set 6 (due April 10)

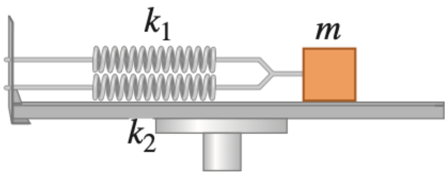
1. Consider a rocket in space traveling through a substance such that the only force acting on the rocket is the force of linear drag. Assume that the fuel is ejected at a constant speed u. The initial mass of the rocket is m_0 and the change in the mass is constant so that $dm/dt = -\alpha$.

   (a) (1pts) Use the equations of motion to deduce the terminal velocity of the rocket.

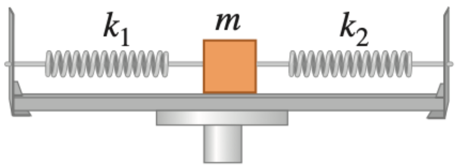
   (b) (2pts) Assuming that the rocket starts from rest, find its velocity as a function of the mass.

2. Two springs with the same unstretched length but different spring constants k_1 and k_2 are attached to a block with mass m on a frictionless surface. Calculate the effective spring constant k_{eff} such that $\sum F = -k_{\text{eff}}$ for each of the cases shown in the pictures below.

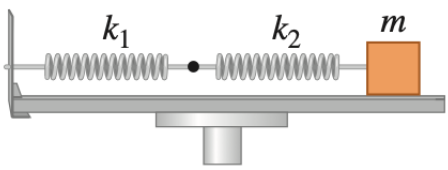
   (a) (1pt)



   (b) (1pt)



   (c) (1pt)



   (d) (1pt) Assume now that you have an object of mass m attached to only one spring with spring constant k and oscillating with frequency f_1. If the spring is cut in half and the same object is attached from one of the two halves, its frequency is now f_2. What is the ratio f_2/f_1?

3. A rocket is fired vertically upward and explodes at a maximum height of $80m$. The rocket breaks into two pieces: one with mass $1.40kg$ and the other with mass $0.28kg$. In the explosion, $860J$ of chemical energy is converted to kinetic energy of the two fragments. Assume air resistance can be ignored.

   (a) (1pt) What is the speed of each fragment just after the explosion?

   (b) (2pt) The two fragments hit the ground at the same time. Assuming that the ground is horizontal, what is the distance between the points on the ground where they land?